Enrollment No: ______________________ Exam Seat No: ______________________

C. U. SHAH UNIVERSITY

Summer Examination-2020

Subject Name : Engineering Mathematics – 3

Subject Code : 4TE03EMT2 **Branch: B. Tech (All)**

Semester : 3 **Date : 25/02/2020** **Time : 02:30 To 05:30** **Marks : 70**

Instructions:

(1) Use of Programmable calculator & any other electronic instrument is prohibited.
(2) Instructions written on main answer book are strictly to be obeyed.
(3) Draw neat diagrams and figures (if necessary) at right places.
(4) Assume suitable data if needed.

Q-1 **Attempt the following questions:** **(14)**

a) The period of $\sin pt$ is

(A) 2π (B) $\frac{2\pi}{p}$ (C) $\frac{\pi}{p}$ (D) none of these

b) In the Fourier series expansion of $f(x) = x^3$ in $(-1, 1)$

(A) only sine terms are present (B) both sine and cosine terms are present
(C) only cosine terms are present (D) constant term is present

c) If $f(x) = x\cos x$ in $(-\pi, \pi)$ then the value of a_n equal to

(A) 0 (B) $\frac{\pi}{2}$ (C) $-\pi$ (D) 2π

d) Laplace transform of $t^{-\frac{1}{2}}$ is

(A) $\frac{\pi}{\sqrt{s}}$ (B) $\sqrt{\frac{\pi}{s}}$ (C) $\frac{\sqrt{\pi}}{s}$ (D) none of these

e) $L^{-1}\tan^{-1}\left(\frac{1}{s}\right)$ is

(A) $\frac{\cos t}{t}$ (B) $\frac{\sin t}{t}$ (C) $\frac{\cot t}{t}$ (D) $\frac{\tan t}{t}$

f) The Laplace transform of $tu\,(t-2)$ is

(A) $\left(\frac{1}{s^2}+\frac{2}{s}\right)e^{-2s}$ (B) $\frac{1}{s^2}e^{-2s}$ (C) $\left(\frac{1}{s^2}-\frac{2}{s}\right)e^{-2s}$ (D) $\frac{1}{s^2}e^{2s}$

g) The C.F. of the differential equation $(D^3+2D^2+D)=x^2$ is

(A) $y=c_1+(c_2x+c_3)e^{2x}$ (B) $y=c_1+(c_2+c_3x)e^{-x}$ (C) $y=c_1+(c_2x+c_3)e^{x}$
(D) none of these

h) The P. I. of $(D^2+1)y=\cosh 3x$ is

(A) $\frac{1}{10}\sinh 3x$ (B) $\frac{1}{5}\cosh 3x$ (C) $\frac{1}{10}\cosh 3x$ (D) none of these

i) $\frac{1}{D-a}X$, (where $X=k$ is constant) equal to

(A) $-\frac{k}{a}$ (B) $\frac{k}{a}$ (C) ka (D) $-ka$

j) Eliminating the arbitrary constants, a and b from $z=(x+a)(y+b)$, the partial differential equation formed is

(A) $z=\frac{p}{q}$ (B) $z=p+q$ (C) $z=pq$ (D) $xq=yp$

k) The general solution of the equation $p\tan x+q\tan y=\tan z$ is

(A) $F\left(\frac{\cos x}{\cos z},\frac{\cos y}{\cos z}\right)=0$ (B) $F(\sin x\ \sin y,\sin x+\sin y)=0$

(C) $F\left(\frac{\sin y}{\sin x},\frac{\sin z}{\sin x}\right)=0$ (D) none of these

l) Particular integral of $(D^2-D'^2)z=\cos(x+y)$ is

(A) $\frac{x}{2}\cos(x+y)$ (B) $x\sin(x+y)$ (C) $x\cos(x+y)$ (D) $\frac{x}{2}\sin(x+y)$

m) The order of convergence in Bisection method is
(A) zero (B) linear (C) quadratic (D) None of these

n) The criterion for convergence for solving $f(x)=0$ by the Newton-Raphson method is

(A) $\left|\{f'(x)\}^2\right|>|f(x)\cdot f''(x)|$ (B) $\left|\{f'(x)\}^2\right|<|f(x)\cdot f''(x)|$

(C) $\left|\{f'(x)\}^2\right|=|f(x)\cdot f''(x)|$ (D) none of these

Attempt any four questions from Q-2 to Q-8

Q-2 **Attempt all questions** **(14)**

a) Using Newton-Raphson method, find the root of $f(x)=\sin x+\cos x$ correct to three decimal places. **(5)**

b) One real root of the equation $x^3-4x-9=0$ lies between 2.625 and 2.75. Find the root using Bisection method. **(5)**

c) Evaluate: $L\left(te^{-4t}\sin 3t\right)$ **(4)**

Q-3 **Attempt all questions** **(14)**

a) Show that $x^2=\frac{\pi^2}{3}+4\sum_{n=1}^{\infty}(-1)^n\frac{\cos nx}{n^2}$ in the interval $-\pi\le x\le\pi$. Hence deduce that $\frac{1}{1^2}-\frac{1}{2^2}+\frac{1}{3^2}-.....=\frac{\pi^2}{12}$. **(5)**

b) Find the Fourier series for **(5)**

$$f(x) = a(x-l), \quad -l < x < 0$$
$$= a(l+x), \quad 0 < x < l$$

and deduce that $1 - \frac{1}{3} + \frac{1}{5} - \frac{1}{7} + ... = \frac{\pi}{4}$.

c) Given that one root of the equation $x^3 - 4x + 1 = 0$ lies between 1 and 2. Find the root correct to 3 significant digits using Secant method. **(4)**

Q-4 **Attempt all questions** **(14)**

a) Using Laplace transform method solve: **(5)**

$$\frac{d^2x}{dt^2} + 2\frac{dx}{dt} + 5x = e^{-t}\sin t, \quad x(0) = 0, \quad x'(0) = 1$$

b) Using convolution theorem, evaluate $L^{-1}\left\{\frac{s}{\left(s^2+4\right)^2}\right\}$. **(5)**

c) Solve: $\frac{\partial^2 z}{\partial x^2} = a^2 z$ given that when $x = 0$, $\frac{\partial z}{\partial x} = a\sin y$ and $\frac{\partial z}{\partial y} = 0$. **(4)**

Q-5 **Attempt all questions** **(14)**

a) Evaluate: $L^{-1}\left[\frac{1}{s^3 - a^3}\right]$ **(5)**

b) Solve: $\left(D^2 - 2D + 1\right)y = xe^x \sin x$ **(5)**

c) Solve: $pz - qz = z^2 + (x+y)^2$ **(4)**

Q-6 **Attempt all questions** **(14)**

a) Solve: $D^2\left(D^2+4\right)y = 48x^2$ **(5)**

b) If $f(x) = \begin{cases} x & , 0 < x < \frac{\pi}{2} \\ \pi - x & , \frac{\pi}{2} < x < \pi \end{cases}$ then show that **(5)**

$$f(x) = \frac{4}{\pi}\left(\frac{\sin x}{1^2} - \frac{\sin 3x}{3^2} + \frac{\sin 5x}{5^2} - .....\right).$$

c) Solve: $L\left(\frac{e^{-at} - e^{-bt}}{t}\right)$ **(4)**

Q-7 **Attempt all questions** **(14)**

a) Solve by the method of variation of parameters: $\frac{d^2y}{dx^2} + a^2 y = \sec ax$ **(5)**

b) Solve: $x^3\frac{d^3y}{dx^3} + 2x^2\frac{d^2y}{dx^2} + 2y = 10\left(x + \frac{1}{x}\right)$ **(5)**

c) Solve: $\frac{\partial^2 z}{\partial x^2} - 4\frac{\partial^2 z}{\partial y^2} = \cos 2x \cos 2y$ **(4)**

Q-8 **Attempt all questions** **(14)**

a) The following table gives the variations of periodic current $t = f(t)$ amperes over a period T sec. **(7)**

t (sec) :	0	$\frac{T}{6}$	$\frac{T}{3}$	$\frac{T}{2}$	$\frac{2T}{3}$	$\frac{5T}{6}$	T
$i\ (A)$:	1.98	1.30	1.05	1.30	– 0.88	– 0.5	1.98

Show, by harmonic analysis, that there is a direct current part of 0.75 amp. in the variable current and obtain the amplitude of the first harmonic.

b) Using the method of separation of variables, solve $\frac{\partial u}{\partial x} = 2\frac{\partial u}{\partial t} + u$, given $u(x,\ 0) = 6e^{-3x}$ **(7)**